Optimization

Satellites are designed to optimize the number of experiments they can carry, while at the same time keeping the mass and power requirements at a minimum. The picture shows the IMAGE satellite with the dark solar cells attached to its surface.

Here is one example of a simple problem that can be encountered by a satellite designer.

A satellite is designed to fit inside the nose-cone (shroud) of a Delta II rocket. There is only enough room for a single satellite, and it cannot have deployable solar panels to generate electricity using solar cells. Instead, the solar cells have to be mounted on the exterior surface of the satellite. At the same time, the satellite configuration is that of a hexagonal prism. The total volume of the satellite is 10 cubic meters. The solar cells will be mounted on the hexagonal top, bottom, and the rectangular side panels of the satellite.

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that maximize the surface area and hence the available power that can be generated by the solar cells?

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems?

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite?

Problem 4 – If the density of the satellite is 1000 kilograms per cubic meter, and the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms)

Answer Key

Problem 1 - If the width of a panel is W, and the height of the satellite is H, what are the dimensions of the satellite that maximize the surface area and hence the available power that can be generated by the solar cells?

Answer:
The volume of a hexagonal prism is given by

$$V = \frac{3\ (3)^{1/2}\ H\ W^2}{2}$$

The surface area is given by

$$A = 6WH + 3\ (3)^{1/2}\ W^2$$

Since V = 10 cubic meters, we can solve for H to get

$$H = \frac{20}{3\ (3)^{1/2}\ W^2}.$$

Eliminate H from the equation for surface area to get

$$A = \frac{40}{3\ (3)^{1/2}\ W} + 3\ (3)^{1/2}\ W^2$$

Differentiate A with respect to W, set this equal to zero.

$$\frac{40}{3\ (3)^{1/2}\ W^3} = 6\ (3)^{1/2}\ W$$

A bit of algebra gives us $W^3 = 40/(3*18)$ so **W = 0.90** meters and from the definition for H we have **H = 4.75 meters**.

Problem 2 - If only 1/2 of the solar cells receive light at any one time, and the power they can deliver is 100 watts per square meter, what is the maximum power that this satellite can provide to the experiments and operating systems? Answer: The total surface area is $A = 6\ (0.9)\ (4.75) + 3\ (3)^{1/2}\ (0.9)^2 = 25.7 + 4.2 = 29.9$ square meters. Only ½ are available for sunlight, and so the total power will be about 29.9 x 0.5 x 100 = **1,495 watts**.

Problem 3 - If the mass of the panels is 3 kg per square meter, what is the total mass of this satellite? Answer: 3 x 29.9 = **89.7 kilograms.**

Problem 4 – If the density of the satellite is 1000 kilograms per cubic meter, and the launch cost is $10,000 per pound, how much will it cost to place this satellite into orbit? (Note, 1 pound = 0.453 kilograms). Answer: The volume of the hexagonal satellite is V = 10 cubic meters, so the mass is 1000 x 10 = 10,000 kilograms or 10 metric tons. The cost to launch is 10,000 kilograms x 10,000 dollars/pound x (1 pound/ 0.453 kg) = $100 million/0.453 = **$220 million dollars.**